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AXIAL FORM FACTORS OF $K_{\ell 4}$ DECAY

M.K.Volkov

The box diagrams and the diagrams with an intermediate axial-vector meson K_1 are shown to be important in estimating the axial form factors of the $K_{\ell 4}$ decay. Calculations have been made in the framework of the quark model of a superconducting type, but conclusions are rather general, valid for any linear sigma-model.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Аксиальные формфакторы $K_{\ell 4}$ распада

М.К.Волков

Показана важная роль, которую играют контактная диаграмма и диаграммы с промежуточными аксиально-векторными мезонами при определении величин аксиальных формфакторов $K_{\ell 4}$ распада. Расчеты проведены в кварковой модели сверхпроводящего типа, однако выводы имеют достаточно общий характер, справедливый и в других кварковых и феноменологических моделях.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

In ref./1/ , we studied the $K_{\ell 4}$ decay form factors in a nonlinear chiral theory, and it was just vector form factor that was there calculated. Estimations made for the axial form factors coincided with the values obtained earlier in current algebra /2/, which was explained by the mere use of the tree approximation. It is known that the total probability of $K_{\ell 4}$ decay computed with those values of the axial form factors appears to be one-half the experimental value. In ref./1/ it is pointed out that inclusion of the box baryon diagrams can increase these form factors by 30-40%.

In this note, a more thorough analysis will be given for all the diagrams that significantly contribute to the axial form factors of $K_{\ell 4}$ decay in the framework of a quark model. The results do not pretend to a high quantitative accuracy (it amounts to within 30% for every diagram and is typical of the results of an approximate chiral SU(3)x xSU(3) symmetry). However, a comparative analysis of contributions from various diagrams provides us with a rather confident picture that may be considered to be weakly dependent on a particular model.

The $K_{\ell 4}$ decay amplitude is of the form

$$T_{K_{\ell 4}} = [f(p_+ + p_-)^\mu + g(p_+ - p_-)^\mu + r(p_K - p_+ - p_-)^\mu + ih\epsilon^{\mu\nu\sigma\rho}(p_K)_\nu \times \times (p_+ + p_-)_\sigma (p_+ - p_-)_\rho] \times \ell_\mu^{(-)} K^+ \pi^+ \pi^-, \tag{1}$$

where p_K, p_+ and p_- are momenta of K, π^+ and π^- mesons, respectively, $\ell_\mu^{(-)} = \frac{G}{\sqrt{2}} \sin\theta_c \, \bar{u}_\nu \gamma_\mu (1 + \gamma_5) u_\ell$ is the leptonic current, θ_c is the Cabibbo angle, f, g, r are axial form factors, and h is a vector form factor. The $K_{\ell 4}$ decay probability depends essentially on the form factors f and g to be calculated in this note.

To compute the probability of the decay $K^+ \to \pi^+ \pi^- \bar{e} \nu$, we take advantage of the approximate formula[3]:

$$w_{K^+ \to \pi^+ \pi^- \bar{e} \nu} = \frac{G^2 m_K^7 \sin^2\theta_c}{360(4\pi)^5} (0.03 f^2 + 0.006 g^2) . \tag{2}$$

Then, with the standard values of the form factors f and g

$$f = g = \frac{1}{\sqrt{2} F_\pi} \quad (F_\pi = 93 \text{ MeV}), \tag{3}$$

following from current algebra[2] and from nonlinear chiral theories[1], we arrive at the $K_{\ell 4}$ decay probability*

$$w_{K^+ \to \pi^+ \pi^- \bar{e} \nu} = 1.4 \cdot 10^3 \text{ sec}^{-1} \tag{4}$$

that is lower than the experimental value[4],

$$w^{exp}_{K^+ \to \pi^+ \pi^- \bar{e} \nu} = (3.15 \pm 0.12) \cdot 10^3 \text{ sec}^{-1} . \tag{5}$$

Once these preliminary remarks are made, we may proceed to analyse those diagrams that produce dominant contributions to the form factors f and g. We will estimate these diagrams using the Lag-

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**In ref.*[1], *a somewhat higher value was obtained with (3)*

$$w_{K^+ \to \pi^+ \pi^- \bar{e} \nu} = 1.7 \cdot 10^3 \text{ sec}^{-1} . \tag{4'}$$

rangians of a superconducting-type quark model[5]:

$$\mathfrak{L} = \bar{q}[ig_i\gamma_5\lambda_i\phi_i + \tilde{g}_i\lambda_i\sigma_i + \frac{g_\rho}{2}\lambda_i(\hat{V}_i + \gamma_5\hat{A}_i)]q. \qquad (6)$$

Here $\bar{q} = (\bar{u}, \bar{d}, \bar{s})$ are quark fields with three colours, λ_i are Gell-Mann matrices ($0 \le i \le 8$), ϕ_i, σ_i, V_i^ν and A_i^ν are nonets of pseudoscalar, scalar, vector and axial-vector fields, g_ρ is a vector coupling constant ($g_\rho^2/4\pi \approx 3$),

$$\tilde{g}_i = \frac{g_\rho}{\sqrt{6}}(1 \le i \le 3), \quad g_i = \tilde{g}_i Z^{1/2} = \frac{m_i}{F_{\phi_i}} \quad (Z = (1 - \frac{6m_u^2}{m_{a_1}^2})^{-1} \approx 1.4),$$

m_i are masses of constituent quarks, m_{a_1} is the mass of the axial-vector meson (the constant Z is a result of inclusion of $\phi_i \to a_i$ transitions), $g_{1,2,3} = m_u/F_\pi$, $g_K = (m_u + m_s)/2F_K$, $F_K = 1.16\, F_\pi$, $m_u = 280$ MeV, $m_s = 460$ MeV. Divergent quark loops are cut off at an energy of $\Lambda = 1.25$ GeV that characterizes the region where spontaneous chiral symmetry breaking occurs and constituent quarks appear with the above-indicated masses (together with a quark condensate). Inclusion of divergent quark diagrams leads to phenomenological chiral Lagrangians of mesons. Finite quark loop diagrams give futher contributions that are not contained in the basis chiral Lagrangians (for instance, the Wess-Zumino terms describing decays $\pi^0 \to \gamma\gamma, \omega \to 3\pi, ...$).

First consider the diagrams with intermediate scalar mesons σ_u and σ_K (see Fig. 1a,b). Each of these diagrams consists of two divergent quark loops, therefore the corresponding vertices enter into the phenomenological chiral Lagrangians[5,6]:*

$$\mathfrak{L}_1 = \frac{g_\rho}{\sqrt{2}} K_{1\nu}^-(\pi^+\partial_\nu\sigma_K - \sigma_K\partial_\nu\pi^+) + \frac{g_\rho}{2} K_{1\nu}^-(K^+\partial_\nu\sigma_u - \sigma_u\partial_\nu K^+), \qquad (7)$$

$$\mathfrak{L}_2 = 4m_u g_\pi Z^{1/2}\sigma_u\pi^+\pi^- + 2\sqrt{2}\, m_s g_\pi Z^{1/2}\sigma_K\pi^+K^-. \qquad (8)$$

In (7) the vertex with an axial-vector meson is included instead of the vertex with a lepton pair $\bar{e}\nu$. To come back to the lepton current,

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These Lagrangians may be obtained from (6) by the method given in [5]

$$\sigma_u = \frac{\sqrt{2}\,\sigma_0 + \sigma_8}{\sqrt{3}}, \quad \sigma_K = \frac{\sigma_6 - i\sigma_7}{\sqrt{2}}.$$

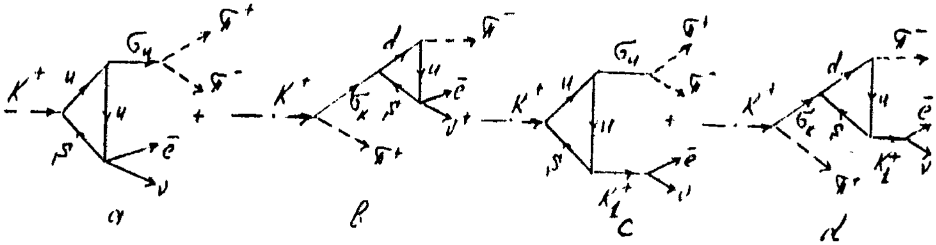

Fig.1

we should make the change: $\frac{g_\rho}{2}\sqrt{2}\,K^{-}_{1\nu}\to \ell^{(-)}_{\nu}$. As a result, we obtain the following contributions to the $K_{\ell 4}$ decay amplitude

$$T^{(\sigma)}_{K_{\ell 4}} = i\sqrt{2}\,2Z^{1/2} g_\pi \Big\{ \frac{m_u[(p_K - p_+ - p_-)^\nu + 2(p_+ + p_-)^\nu]}{m^2_{\sigma_u} - (p_+ + p_-)^2} - \frac{m_s}{m^2_{\sigma_K} - (p_K - p_-)^2} \times \tag{9}$$

$$\times [(p_K - p_+ - p_-)^\nu + (p_+ + p_-)^\nu - (p_+ - p_-)^\nu]\Big\} \ell^{(-)}_\nu K^+ \pi^+ \pi^-.$$

From (9), in the limits $m^2_{\sigma_u}\to\infty$ and $m^2_{\sigma_K}\to\infty$, expression (3) can easily be obtained for the form factors f and g. For this purpose it suffices to make use of the formulae $m^2_{\sigma_u} = 4m^2_u + m^2_\pi$, $m^2_{\sigma_K} = 4m_u m_s + m^2_K$ and tend the masses m_u and m_s to infinity ($m_u = m_s = \infty$, $Z = 1$). So it may be verified that for the results of current algebra only those diagrams are responsible which are drawn in Figs.1a, b (for the form factors f and g).

Estimation of the contribution of amplitude (9) to the form factor f requires consideration of a large width of the scalar meson σ_u, we will identify with the scalar resonance ϵ (700). Its width is close to the mass of the meson. The width of the meson σ_K which may be identified with the resonance κ(1350) (or K^*_0(1350) in new terms) can be neglected. The effective values of momenta, $(p_+ + p_-)^2 \simeq 0.45\,m^2_K$ and $(p_K - p_-)^2 \simeq 5.5 m^2_\pi$, can be found from phase integrals. With all that taken into account we get for the form factor f *

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* *Similar estimations were successfully used for the description of the decays $K_S\to 2\gamma$, $K\to 2\pi$ and interpretation of the rule $\Delta T = 1/2$ (see ref.* /7/ *). In ref.* /8/ *an attempt was made to take account of the one-loop meson approximation in such estimates.*

$$f^{(\sigma)} = \frac{4Z^{1/2}}{\sqrt{2}F_\pi}\left\{\frac{2m_u^2}{m^2_{\sigma_u}-(p_+ + p_-)^2}\left[1+\frac{\sqrt{(p_+ + p_-)^2}\,\Gamma_{\sigma_u}((p_+ + p_-)^2)}{m^2_{\sigma_u}-(p_+ + p_-)^2}\right]^{-1/2} - \frac{m_u m_s}{m^2_{\sigma_K}-(p_K - p_-)^2}\right\} = \frac{1.15}{\sqrt{2}F_\pi} \quad (10)$$

The width $\Gamma_{\sigma_u\to 2\pi}(q^2)$ is calculated by the formula

$$\Gamma_{\sigma_u \to 2\pi} = \frac{3}{2}\,\frac{Z}{\pi\sqrt{q^2}}\left(\frac{m_u^2}{F_\pi}\right)^2\sqrt{1-\frac{(2m_\pi)^2}{q^2}}. \quad (11)$$

Till now only standard contributions were considered to the form factor f studied by many authors and, in particular, in recent paper /8/ . Let us now apply to the study of those diagrams which are usually beyond the field of vision, but, as will be shown below, are rather important in the determination of the form factors f and g.

In ref. /9/ it is shown that the experimentally observed inequality $h_A < h_V$ in the decay $\pi^- \to \bar{e}\nu\gamma$ may be associated with an extra diagram with an intermediate axial-vector a_1 meson between a lepton pair $e\bar{\nu}$ and vertex $\pi\gamma a_1$ (h_V and h_A are vector and axial-vector form factors of the decay $\pi^- \to e\bar{\nu}\gamma$). Without that diagram, quark models gave equal values for the form factors h_V and h_A ($h_V = h_A = 1/8\pi^2 F_\pi$). The diagram with the intermediate a_1 meson induced an extra factor $Z^{-1} \simeq 0.7$ in the form factor h_A ($h_A = h_V Z^{-1}$).

Making analogous calculations for the $K_{\ell 4}$ process it can be shown that the inclusion of diagrams with an intermediate K_1^+ meson (Figs.1c,d) also results in the extra factor Z^{-1} in expression (10) for the form factor $f^{(\sigma)}$. As a result, we get*

$$\tilde{f}^{(\sigma)} = f^{(\sigma)} Z^{-1} = \frac{0.82}{\sqrt{2}F_\pi}. \quad (12)$$

Aside from the diagrams with intermediate scalar mesons, also important for the computation of the form factor f are box diagrams depicted in Fig. 2 (contact terms). Diagrams of that sort are always taken

We assume that the factors Z^{-1} for both strange and nonstrange particles are approximately equal to each other, $Z_K^{-1} \simeq Z_\pi^{-1} = (1 - \frac{6m_u^2}{m_{a_1}}) \simeq 0.7$. If the mass difference in this expression is taken into account between strange and nonstrange quarks, one should also take account of the quark-mass difference in loop integrals, and there will occur compensation of both the effects (see /9/).

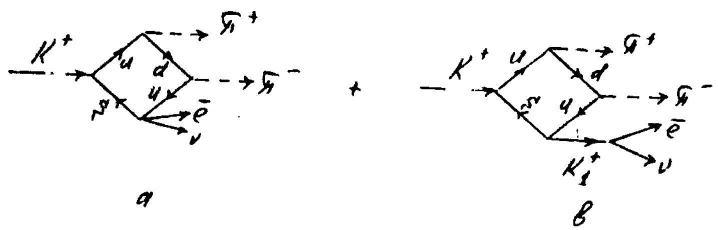

Fig.2

into consideration when describing $\pi\pi$ and πK scattering, meson polarizability, etc.[5]. In all those processes the box diagrams are connected with divergent integrals, therefore the corresponding terms enter into the basic phenomenological Lagrangians of the sigma-model. In case of the $K_{\ell 4}$ decay, the box diagrams are related to convergent integrals; therefore no analogous terms are present in the phenomenological Lagrangians, and estimations of the form factors made in current algebra or in nonlinear chiral Lagrangians do not usually take account of those contributions. These diagrams also have not been taken into consideration in ref.[8], although they give rather significant contributions to the form factors we are interested in. With Lagrangian (6), we may obtain the following estimate for diagram, Fig.2a:

$$T^{(\square)} = i\frac{3m_u(m_u+m_s)}{\sqrt{2}\,8\pi^2 F_\pi^2 F_K}[(p_+ + p_-)^\nu + (p_+ - p_-)^\nu + 2(p - p_+ - p_-)^\nu]\ell_\nu^{(-)} K^+\pi^+\pi^-. \quad (13)$$

The diagram with an intermediate K_1^+ meson (Fig.2b) diminishes the amplitude (13) Z times, and for these diagrams the form factors f and g take the values:

$$\tilde{f}^{(\square)} = \tilde{g}^{(\square)} = \frac{3m_u(m_u+m_s)}{\sqrt{2}\,8\pi^2 F_\pi^2 F_K Z} = \frac{0.56}{\sqrt{2}\,F_\pi}. \quad (14)$$

It is easy to see that this value is consistent with estimate (12)*.

**We do not here evaluate the corrections to (14) for the* q^2*-terms. They may produce an increase of value (14) within 30% (the K meson mass taken into account).*

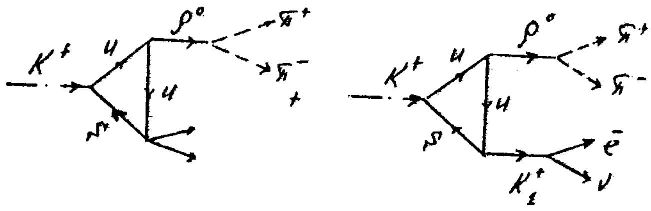

Fig.3

Now let us proceed to estimation of contributions to the form factor g from diagrams drawn in Fig.3. These diagrams are again connected with divergent integrals, therefore the corresponding vertices can be found in the phenomenological chiral Lagrangians[5,8] :

$$\mathcal{L}_3 = -i g_\rho^2 F_K Z \rho_0^\nu K^+ \sqrt{2}\, \ell_\nu^{(-)},$$
$$\mathcal{L}_4 = i g_\rho \rho_0^\nu (\pi^+ \partial_\nu \pi^- - \pi^- \partial_\nu \pi^+) = g_\rho (p_+ - p_-)^\nu \rho_0^\nu \pi^+ \pi^- . \qquad (15)$$

As a result, the diagram 3a leads to the following amplitude:

$$T^{(\rho)} = i\sqrt{2}\, g_\rho^2 F_K Z \frac{f_\rho((p_+ + p_-)^2)}{m_\rho^2 - (p_+ + p_-)^2} (p_+ - p_-)^\nu \ell_\nu^{(-)} K^+ \pi^+ \pi^- . \qquad (16)$$

$$\left(f_\rho(q^2) = 1 - \frac{m_\rho^2 - q^2}{8\pi^2 F_\pi^2} \right)$$

Here $f_\rho(q^2)$ is form factor of the $\rho \to 2\pi$ decay (see [10]). Upon inclusion of diagram 3b with the intermediate K_1 meson the factor Z in (16) disappears, and the form factor $\tilde{g}^{(\rho)}$ is defined by the expression:

$$\tilde{g}^{(\rho)} = \frac{\sqrt{2}\, g_\rho^2 F_K}{m_\rho^2 - 0.45 m_K^2} \left[1 - \frac{m_\rho^2 - 0.45 m_K^2}{8\pi^2 F_\pi^2} \right] = \frac{0.49}{\sqrt{2}\, F_\pi} . \qquad (17)$$

Using the results of ref[9] we may estimate the contribution of q^2 terms to amplitude (16). They produce a 30% increase in $g^{(\rho)}$. On the other hand, the contribution of the ρ meson width will decrease this value. Assuming that all these effects lead to mutual compensations and their contributions amount to about 30%, we shall not take them into consideration.

Combining all the obtained estimates, we arrive at the following results

$$f = f^{(\sigma)} + f^{(\Box)} = \frac{0.82 + 0.56}{\sqrt{2}\,F_\pi} \simeq \frac{1.4}{\sqrt{2}\,F_\pi}, \qquad (18)$$

$$g = g^{(\sigma)} + g^{(\Box)} + g^{(\rho)} = \frac{0.26 + 0.56 + 0.49}{\sqrt{2}\,F_\pi} \simeq \frac{1.3}{\sqrt{2}\,F_\pi}. \qquad (19)$$

Then, from formula (2) for the total probability of the decay $K^+ \to \pi^+\pi^-\bar{e}\nu$ we get

$$w_{K^+ \to \pi^+\pi^-\bar{e}\nu} = 2.7 \cdot 10^3 \ \mathrm{sec}^{-1}, \qquad (20)$$

which is within the limits of our accuracy, consistent with the experimental value (5).

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